Simple parametrization of α-decay spectroscopic factor in $150 \leq A \leq 200$ region

G. Gangopadhyay

Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata-700 009, India

Abstract. Half life values for α-decay in the mass region $A = 150 - 200$ have been calculated in the Microscopic Super Asymmetric Fission Model. The interaction between the α-particle and the daughter nucleus has been obtained in the double folding model with the microscopic interaction DDM3Y1. Theoretical densities for the nuclei involved have been calculated in Relativistic Mean Field approach with the Lagrangian density FSU Gold. Spectroscopic factors for α-decay, calculated as the ratios of the theoretically calculated and experimentally measured half life values, show a simple dependence on the mass number and the product of the number of valence protons and neutrons. The implications of the parameterization have been discussed. Finally, based on this simple relation, α-decay half life values in a number of nuclei have been predicted.

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1. Introduction

Studies on α-decay have often been undertaken in heavy and superheavy mass regions to investigate the nuclear baryon density profile. Both α-decay and cluster decay are known to take place through tunnelling of the potential barrier which may theoretically be constructed from the densities of the daughter nucleus and the lighter decay particle using a suitable interaction. The analytical super asymmetric fission model was developed to describe this process[1]. In the present work, we employ the Microscopic Super Asymmetric Fission Model[2] to calculate the tunnelling probability in the WKB approximation. The potential between the α-particle and the daughter nucleus is obtained microscopically in the double folding model by folding the proton and neutron densities in the α-particle and the daughter nucleus with some suitable interaction. The densities may be obtained either from some phenomenological recipe or from a theoretical calculation. In the present work we utilize the second approach and use the microscopic densities obtained from Relativistic Mean Field (RMF) theory. This method has the advantage that it can be extended to nuclei far from the stability valley where phenomenological densities are not applicable.

In a microscopic approach, the spectroscopic factor is the ratio of the experimental decay constant and the decay constant calculated in a simple Gamow picture. The experimental decay constant is given by

$$\lambda = \nu S P \tag{1}$$

where ν is the assault frequency, P is the tunnelling probability and S is the spectroscopic factor. If we consider the α particle to be preformed at the surface, the theoretical results may be obtained by the product of the first two terms. Thus the spectroscopic factor in α-decay was introduced to incorporate the preformation probability of α-particle. It may be considered as the overlap between the actual ground state configurations of the parent and the configuration described by one α-particle coupled to the ground state of the daughter. In this interpretation, it is expected to be less than unity as the probability of α-particle already preformed at the nuclear surface is small. It has been shown[3] in cluster decay that for a daughter nucleus, close to the magic number, the spectroscopic factor scales as $S_\alpha^{(A-1)/3}$ where A is the mass of the cluster and S_α is the spectroscopic factor for alpha decay.

The spectroscopic factor for α-decay is seen to depend on the odd-even effect as its value for an even-even daughter is seen to be nearly double than that for an odd-mass daughter[3]. In nuclei spread over a large mass region, it should incorporate other nuclear structure effects like deformation, shell closure, odd-even effects, etc. It is useful to parametrize the spectroscopic factor in terms of various simple quantities. Such parametrizations were obtained for various other nuclear quantities in terms of suitable functions of valence neutron and proton numbers (N_n and N_p, respectively)[4]. Particularly, we find that quantities such as B(E2) values, which are measures of deformation can be parametrized easily in the N_pN_n scheme We have already studied α-decay in heavy nuclei[5] and have have seen that the α-decay spectroscopic factor

varies smoothly with such a suitable function. In the present work, we extend our work on α-decay to a lighter mass region, *i.e.* between $A = 150$ and 200. Our aim is to obtain a simple phenomenological expression which can predict the spectroscopic factor in this mass region.

2. Theory

RMF is now a standard approach in low energy nuclear structure. It can describe various features of stable and exotic nuclei including ground state binding energy, shape, size, properties of excited states, single particle structure, features of exotic nuclei, etc[6]. Being based on Dirac phenomenology, it naturally includes the spin degrees of freedom. As mass regions far from the stability valley show indications of spin-orbit quenching, RMF is extremely suited for investigation of these nuclei. There are different variations of the Lagrangian density and also a number of different parameterizations in RMF. We employ a recently proposed Lagrangian density [7], FSU Gold, which involves self-coupling of the vector-isoscalar meson as well as coupling between the vector-isoscalar meson and the vector-isovector meson. This Lagrangian density has earlier been employed to obtain the proton nucleus interaction to successfully calculate the half life for proton radioactivity[8] as well as our earlier works on α and cluster radioactivity[5, 9, 10]. In this work also, we have employed FSU Gold.

Since the nuclear proton and neutron densities as a function of radius are of importance in our calculation, the equations have been solved in co-ordinate space. The strength of the zero range pairing force is taken as 300 MeV-fm for both protons and neutrons. We have assumed spherical symmetry. The microscopic density dependent M3Y interaction, obtained from a finite range nucleon-nucleon interaction by introducing a factor dependent on the nuclear density, is a class of interactions that are frequently used in calculating nucleus-nucleus interaction. In the present work, we have employed the interaction DDM3Y1 which has an exponential density dependence

$$v(r, \rho_1, \rho_2, E) = C(1 + \alpha \exp(-\beta(\rho_1 + \rho_2))) \times (1 - 0.002E)u^{M3Y}(r) \qquad (2)$$

used in Ref. [11] to study α-nucleus scattering with the standard parameter values, *viz.* $C = 0.2845$, $\alpha = 3.6391$ and $\beta = 2.9605$ fm^3. Here ρ_1 and ρ_2 are the densities of the α-particle and the daughter nucleus, respectively and E is the energy per nucleon of the α-particle in MeV. DDM3Y1 uses the direct M3Y potential $u^{M3Y}(r)$ based on the G-matrix elements of the Reid[12] NN potential. The weak energy dependence was introduced[13] to reproduce the empirical energy dependence of the optical potential. The NN interaction has been folded with the theoretical densities of α-particle and the daughter nucleus in their ground states using the code DFPOT[14] to obtain the potential between them. The barrier tunnelling probability for the α-particle has been calculated in the WKB approximation. The assault frequency has been calculated from the decay energy following Gambhir *et al*[15].

In the WKB approximation, the Q-value occurs in the exponential. Consequently, a small change in Q-value can lead to an order of magnitude change in the estimates of

half life and theoretically calculated Q-values do not achieve the desired high accuracy. Following the usual practice, the Q-values (and the decay energies) have been taken from experiment and are from Ref. [16]. Unless later measurements are available, the experimental partial half life values for α-decay have been computed from the compilation by Akovali[17]. The newer measurements are taken from the NNDC website [18].

3. Results

The value of spectroscopic factors are generally expected to be less than unity as the ground state of the parent contain contributions from many other configurations other than the one with the α-particle already formed inside the nucleus and coexisting with the daughter in its ground state. We calculate its value as the ratio of the calculated half life to the experimentally observed value. The logarithms of spectroscopic factors are presented in columns marked 'I' in Table 1.

The fact that the spectroscopic factors incorporates the effect of structure can be seen from the effect of shell closure. For example, in Fig. 1 we plot the $\log_{10} S$ as a function of parent proton number for different isotones. It is clearly seen that despite the fluctuation in the value, except in the case of $N = 110$, in all the nuclei up to

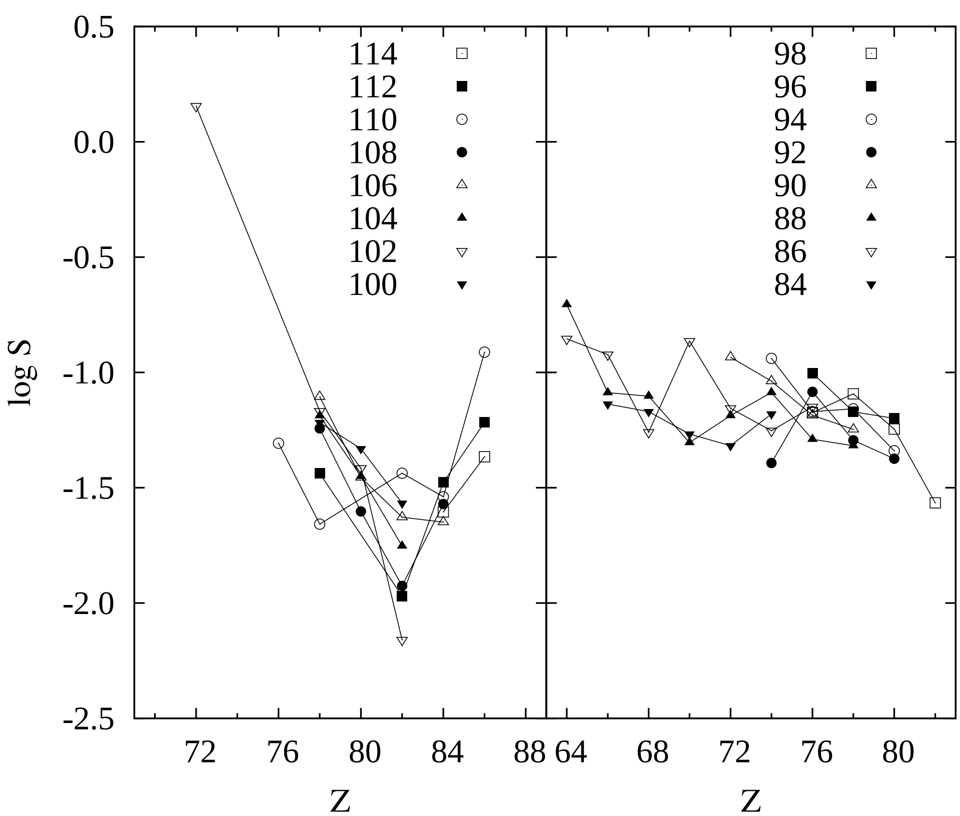

Figure 1. Spectroscopic factors for α-decay as a function of parent proton number. See text for details.

Table 1. Spectroscopic factors (S) for α-decay. See text for details.

Parent	N	log(S)		N	log(S)	
(Z)		I	II		I	II
Rn(86)	114	-1.366	-1.479	112	-1.217	-1.397
	110	-0.912	-1.316			
Po(84)	116	-1.706	-1.699	114	-1.605	-1.649
	112	-1.477	-1.599	110	-1.539	-1.549
	108	-1.571	-1.499	106	-1.649	-1.449
Pb(82)	112	-1.970	-1.800	110	-1.437	-1.782
	108	-1.926	-1.763	106	-1.628	-1.745
	104	-1.753	-1.726	102	-2.160	-1.708
	100	-1.567	-1.689	98	-1.566	-1.670
Hg(80)	108	-1.603	-1.462	106	-1.456	-1.412
	104	-1.451	-1.362	102	-1.416	-1.375
	100	-1.331	-1.387	98	-1.246	-1.400
	96	-1.199	-1.413	94	-1.340	-1.426
	92	-1.374	-1.439			
Pt(78)	112	-1.438	-1.323	110	-1.658	-1.242
	108	-1.243	-1.160	106	-1.106	-1.079
	104	-1.188	-0.997	102	-1.168	-1.042
	100	-1.217	-1.086	98	-1.093	-1.130
	96	-1.171	-1.175	94	-1.158	-1.219
	92	-1.294	-1.263	90	-1.247	-1.308
	88	-1.318	-1.352			
Os(76)	110	-1.307	-0.972	98	-1.174	-0.860
	96	-1.003	-0.936	94	-1.170	-1.012
	92	-1.084	-1.087	90	-1.186	-1.163
	88	-1.290	-1.239	86	-1.149	-1.315
W(74)	94	-0.939	-0.804	92	-1.393	-0.912
	90	-1.038	-1.019	88	-1.087	-1.126
	86	-1.253	-1.233	84	-1.180	-1.340
Hf(72)	102	0.155	-0.043	90	-0.934	-0.875
	88	-1.187	-1.013	86	-1.156	-1.152
	84	-1.318	-1.290			
Yb(70)	88	-1.304	-0.900	86	-0.864	-1.070
	84	-1.267	-1.240			
Er(68)	88	-1.103	-0.787	86	-1.259	-0.989
	84	-1.170	-1.190			
Dy(66)	88	-1.088	-0.675	86	-0.923	-0.908
	84	-1.137	-1.140			
Gd(64)	88	-0.706	-0.750	86	-0.854	-0.952

$N = 100$, $\log_{10} S$ shows a drop. In lighter nuclei, though the values for the shell closure are not available in most cases, we see that the values tend to decrease as one moves towards $Z = 82$. Spectroscopic factors do tend to decrease in general as one moves from high N values towards $N = 82$, but the effect is less prominent.

Besides the effect of shell closure, the spectroscopic factors in the mass region show a weak downward trend with mass number. There are differences for the same mass number corresponding to different Z (and N) values. In an earlier calculation, we observed that the effect of different Z and N values may be expressed as a simple function of number of valence protons and neutrons[8]. This should also take care of the dependence on shell closure. In the present instance we find that the results for the logarithm of the spectroscopic factors can be very easily expressed as a simple function

$$\log_{10} S_p = aA + bN_pN_n \tag{3}$$

where A is the mass number of the parent and N_p and N_n are the number of valence protons and neutrons, respectively. The number of valence particles has been calculated using $Z = 50$ and 82 as proton, and $N = 82$ and 126 as neutron magic numbers. For the nuclei studied in the present work, the χ^2 for the fit is 308. However, more than half the contribution comes from just six nuclei, ^{196,198}Rn, ^{196,190}Po, ^{192}Pb, and ^{188}Pt. The parameters have thus been extracted without taking these nuclei in to account leading

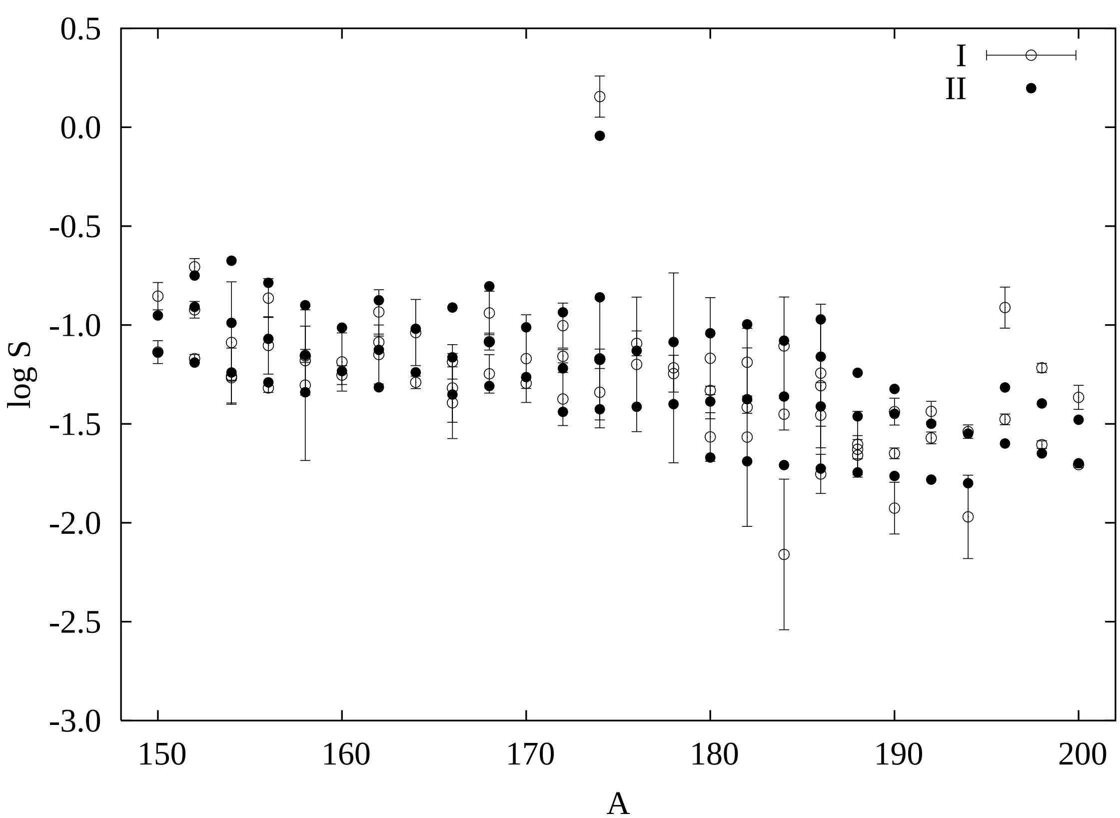

Figure 2. Spectroscopic factors for α-decay. Here I and II refer to the corresponding values in Table 1.

to a χ^2 value of 99.0 for 63 nuclei. They are $a = -0.00928(8)$ and $b = 0.00786(37)$. The values for $\log_{10} S_p$, calculated with the fitted parameters, are tabulated in Table 1 in the columns marked 'II'. We also plot in Fig. 2, the logarithm of the spectroscopic factors (S) calculated using RMF with errors and the values of S_p obtained from eqn. (3) with the fitted parameters. The errors include contributions from uncertainties in half life measurements and branching ratios as well as errors in the theoretical values due to the uncertainty in Q_α value.

We should mention that of the six nuclei which had to be neglected in the fitting procedure, the adopted value for ^{196}Po may actually have a larger error than estimated. The branching ratio to α-decay, 98%, is only an approximate value and may actually be substantially different. For example, Wauters *et al*[19] have measured it to be 94(5)%. Thus it is possible that the estimated error for the α-decay partial half life has a much larger error.

What is the significance of the dependence on N_pN_n? We have already mentioned that simplified parametrization of various nuclear quantities may be obtained if the quantities are plotted as functions of N_p and N_n. We have, in an earlier work, shown that the spectroscopic factor increases with the Casten factor $N_pN_n/(N_p + N_n)$ [8]. Basically the product N_pN_n is related to the integrated n-p interaction strength. Away from a closed shell nucleus, the integrated strength increases and is parametrized by the product N_pN_n. Clearly this also incorporates the shell effects through the number of valence particles. A positive value of the parameter 'b' is thus consistent with our earlier observation in the mass region $A > 208$. Essentially this also includes the effect of deformation, a degree of freedom not included in the present RMF calculation. The fact that the N_pN_n scheme can take deformation into account has been observed in many instances[4].

How may one interpret the negative value for the parameter 'a'? As the mass number increases, the number of possible configurations contributing to the ground state also increases. Thus we may consider the contribution of a particular configuration, in this case that of the α-daughter configuration, to the ground state of the parent nucleus should tend to decrease if all other effects are taken into consideration. In the present calculation, the effects of the shell structure and deformation have been incorporated in the factor N_pN_n. The negative value of the coefficient of A expresses the above mentioned decreasing contribution of the α-daughter configuration.

There are several possible α-decaying nuclei in this mass region for which the half life values have not yet been measured though the Q-values are known from mass measurements. We have theoretically calculated the half life in our approach using the spectroscopic values from eqn. (3) with the fitted parameters. We present our results in Table 2. We restrict ourselves to the nuclei whose half life values are calculated to be less than 10^{18} years. Most of these nuclei undergo beta-decay. We have also tabulated the calculated branching ratios, if beta-decay half life is known and the value is at least 10^{-6}. Because of the large errors in the Q-values, the half life in most of these nuclei may be wrong by a factor of two.

Table 2. Calculated α-decay partial half life values and branching ratios for a number of nuclei with positive Q_α values. See text for details.

Nucleus	Q_α (MeV)	$T_{1/2}$	b_α(%)
^{200}Pb	3.158	1.1×10^{16}Yr	
^{198}Pb	3.718	1.4×10^{10}Yr	
^{196}Pb	4.225	7.5×10^{5}Yr	
^{192}Hg	3.387	8.0×10^{11}Yr	
^{190}Hg	4.069	2.9×10^{5}Yr	
^{188}Po	8.082	18.4 ms	100?
^{184}Os	2.963	1.3×10^{13}Yr	100
^{182}Os	3.382	2.7×10^{8}Yr	
^{180}Os	3.857	1.1×10^{4}Yr	
^{180}W	2.508	1.8×10^{17}Yr	100
^{178}Pb	7.790	4.2 ms	100?
^{178}Os	4.256	14.5Yr	6×10^{-5}
^{178}W	3.005	3.8×10^{10}Yr	
^{176}Os	4.574	55 days	5×10^{-5}
^{176}W	3.337	1.5×10^{7}Yr	
^{174}W	3.602	7.7×10^{4}Yr	
^{172}W	3.838	1.2×10^{3}Yr	1×10^{-6}
^{172}Hf	2.746	1.7×10^{12}Yr	
^{170}W	4.141	8.9Yr	8×10^{-5}
^{170}Hf	2.910	2.6×10^{10}Yr	
^{168}Hf	3.237	1.1×10^{7}Yr	
^{166}Hf	3.548	2.1×10^{4}Yr	
^{164}Hf	3.923	29Yr	1×10^{-6}
^{164}Yb	2.611	6.5×10^{12}Yr	
^{162}Yb	3.047	1.2×10^{8}Yr	
^{160}Yb	3.618	4.0×10^{2}Yr	2×10^{-6}
^{158}Er	2.669	5.1×10^{10}Yr	
^{150}Er	2.296	1.6×10^{15}Yr	

Very little information is available on the nuclei in Table 2. The two nuclei ^{188}Po and ^{178}Pb have not yet been studied experimentally. However, the large Q-value, and hence the short half life, indicates that α-emission will be the dominant, if not the only, decay mode. Experiments suggest that the half life in ^{180}W is $1.1^{+8}_{-4} \times 10^{17}$year, a value in agreement with our calculation. The branching ratio in ^{190}Hg has a upper limit 3.4×10^{-7}% while our calculated value is 1.3×10^{-8}%. A probable candidate for double beta decay, ^{184}Os is known to have a half life greater than 5.6×10^{13}year. Our calculation suggests a slightly smaller value. This prediction may be relatively easier to verify than the other ones as α-decay is expected to the dominant mode in this nucleus

and the half life is comfortably within present measurement capabilities.

4. Summary

Theoretical valued for α-decay half life in the mass region $A = 150 - 200$ have been calculated in the Microscopic Super Asymmetric Fission Model with the theoretical densities obtained from RMF calculations using DDM3Y1 microscopic interaction. Spectroscopic factors for α-decay, calculated as the ratios of the theoretically calculated and experimentally measured half life values, show a simple dependence on the mass number and the product of the number of valence protons and neutrons, the latter being a measure of integrated n-p interaction strength. The mass number dependence is related to the contribution of the daughter plus α particle configuration to the ground state of the parent. Finally, based on this simple relation, we have predicted the half life values for α-decay in a number of nuclei.

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